

GOOD AND BAD TRAINING SEQUENCES FOR ZERO IF SAMPLING EDGE RECEIVERS

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ABSTRACT

In the mobile communication standard GSM/EDGE, the base station can select one of eight training sequence codes as midamble of the downlink transmitted bursts. If the receiving mobile station uses zero intermediate frequency (IF) sampling the channel estimation is sensitive to the DC offset and IQ gain/phase imbalance of the RF transceiver. This note shows for a common class of channel estimators that the sensitivity depends on the selected training sequence code. This sensitivity can become significant for 8PSK modulation.

1. INTRODUCTION

Recently the 2G standard GSM was enhanced by EDGE. In [1] a good overview on EDGE equalization concepts and suggested reading on the EDGE standard are given. The focus of this paper is the training sequence code (“tsc”) based channel estimation which is done before equalization. We restrict ourselves to zero IF sampling receivers, which are most common for EDGE mobile stations.

GSM originally used GMSK modulation and robust channel coding only. These modulation and coding schemes were designed for signal to interference and noise ratios (SINR) of about 9 dB (200 kHz equivalent noise bandwidth), and the bit error rate of the receiver was mainly limited by the noise figure of the RF transceiver. To achieve higher data rates in case of good physical channels, GPRS and later EDGE were introduced. In contrast to GPRS, EDGE also uses 8PSK modulation. Both apply coding schemes with various levels of redundancy. Consequently, for channels with higher SINR up to 30 dB, higher modulation (three bits per symbol) and low redundancy allow higher data rates. In this operating mode besides the RF noise figure other RF impairments like IQ gain/phase imbalance and DC offset play an important role.

The importance of DC offset for EDGE equalizer has also been pointed out in [2]. However, as we will see later, the IQ imbalance can be even more important for the equalizer analysis. Joint channel and DC estimation in connec-

tion with synchronization was considered, e.g., in [3].

In this contribution we show how in high SINR operating modes DC offset and IQ imbalance compromise the channel estimate. The channel estimation is an important step before demodulation of the received burst.

2. MODELING OF THE RECEIVED EDGE BURST

Deriving a complete transmission model for an EDGE burst is not within the scope of this note. A good explanation can be found in several papers, e. g. in [1]. We will focus on that part of the received signal that is used for a non-blind channel estimation in the mobile station for downlink reception. We assume that the signal is symbol space sampled, i.e. the sampling rate is identical to the GSM/EDGE symbol rate $f_T = 13 \text{ MHz}/48$. Moreover, we assume that the symbol-by-symbol rotation of $\phi = \pi/2$ or $\phi = 3\pi/8$ of the base station transmitter is compensated by de-rotation in the digital part of the receiver.

First, we neglect any kind of distortions like noise and RF transceiver impairments. Concerning fading, we assume that the resulting channel impulse response is constant for the short period that is used for the channel estimation of one burst. The baseband IQ samples that are used for channel estimation are represented by a vector \underline{x} and within all these simplifying assumptions we have

$$\underline{x} = \mathbf{T}\underline{h}. \quad (1)$$

The vector \underline{h} represents the channel impulse response including fading, pulse shaping, and all (digital and analog) filters in the transmit and receive path. The order of the FIR filter representing the channel impulse response is denoted by L . Consequently, \underline{h} is an $(L + 1) \times 1$ vector and \underline{x} is an $(N - L) \times 1$ vector. Here, N is the length of the training sequence code, i.e. $N = 26$ for GSM/EDGE normal bursts. \mathbf{T} is a real-valued $(N - L) \times (L + 1)$ Toeplitz matrix whose columns are sub-sequences of the training sequence codes, which are defined in the GSM/EDGE standard [5].

Next, we consider an IQ gain/phase imbalance in the RF transceiver. An IQ imbalance on a complex signal u leading

to an imbalanced complex signal v can be described by:

$$\underline{u} \longrightarrow \underline{v} = \underline{u} + \eta \underline{u}^* \quad (2)$$

Note that $*$ denotes conjugation of the complex components of the vector \underline{u} but does not include transposition. Moreover, we have two degrees of freedom in the description by the normalization of the IQ gain/phase imbalance. Different equations can be obtained by different assumptions for the common gain of I and Q (here $\sqrt{1 + \delta^2 + \varepsilon^2}$) and the common phase shift (here 0). The IQ gain error is given by

$$10\text{dB} \cdot \lg \frac{(1 + \delta)^2 + \varepsilon^2}{(1 - \delta)^2 + \varepsilon^2} \approx 17.4\text{dB} \cdot \delta \quad (3)$$

while the IQ phase error is given by

$$\arctan \frac{2\varepsilon}{1 - \delta^2 - \varepsilon^2} \approx 114.6^\circ \cdot \varepsilon \quad (4)$$

The approximations hold for $\delta^2, \varepsilon^2 \ll 1$.

Since de-rotation is assumed in the digital part of the receiver, the IQ imbalance has to be considered for the rotated received signal $\underline{u} = \mathbf{A}^* \underline{x}$. The de-rotation matrix \mathbf{A} is diagonal

$$\mathbf{A} = \text{diag}\{\underline{a}\} = \text{diag}\{e^{-j0 \cdot \phi} \dots e^{-j(N-L-1) \cdot \phi}\}. \quad (5)$$

To consider IQ imbalance for the de-rotated receive signal we have to replace

$$\underline{x} \longrightarrow \underline{x} + \eta \mathbf{A}^2 \underline{x}^* \quad (6)$$

The next RF impairment that we consider is a DC offset. For zero IF transceivers, a large DC offset is introduced by the RF transceiver. However, it will be compensated partly by digital processing before the de-rotation. This processing is assumed to be “blind”, i.e. no training/pilot symbols are used. The residual DC offset is modeled as a complex random variable d . The variance of d is typically 15 to 25 dB below the wanted signal level. We add the de-rotated DC offset $\underline{a}d$ to our model

$$\underline{x} \longrightarrow \underline{x} + \eta \mathbf{A}^2 \underline{x}^* + \underline{a}d. \quad (7)$$

Note that adding the DC offset before the IQ imbalance leads to a different expression with several DC offset terms, however, they can be summarized to one DC offset again.

The last impairment that is considered is a noise vector \underline{n} . We assume complex white Gaussian noise with a probability density function

$$f_{\underline{n}}(\underline{n}) = \frac{1}{(\pi N_0)^{N-L}} e^{-\frac{1}{N_0} \|\underline{n}\|^2}. \quad (8)$$

Thus, the auto-correlation function $E(\underline{n} \underline{n}^H)$ equals N_0 times the identity matrix. Here and in the following, $\|\cdot\|$ stands for

the complex Euclidean norm and H for transposition plus complex conjugation. The distorted receive signal is

$$\underline{x} = \mathbf{T} \underline{h} + \eta \mathbf{A}^2 \mathbf{T} \underline{h}^* + \underline{a}d + \underline{n}. \quad (9)$$

In the next section, an optimal linear channel estimation method for this signal is derived.

3. CHANNEL ESTIMATION BASED ON THE TRAINING SEQUENCE CODE

Based on Equation (9) an optimum channel estimation can be derived. However, to have an estimator that can be implemented with reasonable effort, we restrict ourselves to linear estimators. An optimum channel estimator would compensate DC offset and IQ imbalance and minimize the error due to noise. However, joint estimation of the unknown imbalance parameter η and the unknown channel impulse response \underline{h} is a nonlinear problem and requires iterative processing.

In the following, we will derive the maximum likelihood channel estimator for $\eta = 0$. The probability density function of the receive signal is

$$\begin{aligned} f_{\underline{x}}(\underline{x}) &= f_{\underline{n}}(\underline{x} - \mathbf{T} \underline{h} - \underline{a}d) \\ &= \frac{1}{(\pi N_0)^{N-L}} e^{-\frac{1}{N_0} \|\underline{x} - \mathbf{T} \underline{h} - \underline{a}d\|^2}. \end{aligned} \quad (10)$$

It is well known that in this situation the joint maximum likelihood estimate for \underline{h} and d is given as the solution of the least square problem

$$\begin{pmatrix} \hat{\underline{h}} \\ \hat{d} \end{pmatrix} = \arg \min_{\underline{h}, d} \left\| \begin{pmatrix} \mathbf{T} & \underline{a} \end{pmatrix} \begin{pmatrix} \underline{h} \\ d \end{pmatrix} - \underline{x} \right\|^2. \quad (11)$$

However, to understand the training sequence dependency of EDGE channel estimation, a different approach is derived.

We obtain the optimum estimation $\hat{\underline{x}}$ by five projection steps. The projection steps are illustrated in Fig. 1. This figure simplifies all subspaces by lines. The subspace spanned by \mathbf{T} is represented by the $\mathbf{T} \mathbf{C}^{L+1}$ -axis. The matrix projecting \underline{x} to the subspace $\mathbf{T} \mathbf{C}^{L+1}$ is (T denotes transposition)

$$\mathbf{P}_{\mathbf{T}} = \mathbf{T}(\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T. \quad (12)$$

However, a projection to the subspace $\mathbf{T} \mathbf{C}^{L+1}$ does not provide the best channel estimate since the DC offset part $\underline{a}d$ is not orthogonal to \mathbf{T} . The DC offset is estimated by the following three projection steps:

Step 1: Projection of \underline{x} to the subspace orthogonal to $\mathbf{T} \mathbf{C}^{L+1}$. The resulting vector

$$\underline{x}'' = \underline{x} - \mathbf{P}_{\mathbf{T}} \underline{x} = \mathbf{Q}_{\mathbf{T}} \underline{x}, \quad (13)$$

has a noise component $\mathbf{Q}_{\mathbf{T}} \underline{n}$ and a DC component $\mathbf{Q}_{\mathbf{T}} \underline{a}d$.

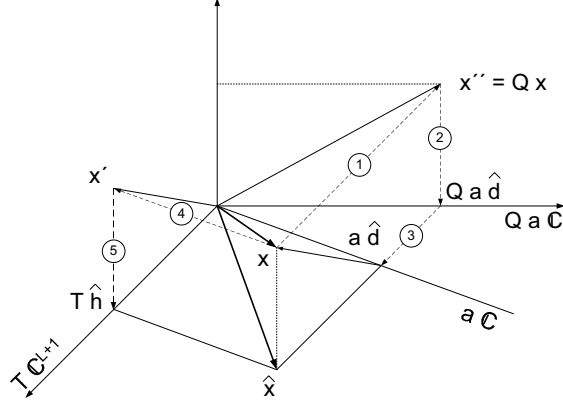


Fig. 1. ML estimation described by projections

Step 2: The DC component is derived by projection again:

$$\underline{x}''_a = \mathbf{P}_{\mathbf{Q}_T \mathbf{a}} \underline{x}'' = \frac{\mathbf{Q}_T \mathbf{a} \mathbf{a}^H \mathbf{Q}_T \underline{x}}{\mathbf{a}^H \mathbf{Q}_T \mathbf{a}}. \quad (14)$$

Note that $\mathbf{P} \cdot \mathbf{P} = \mathbf{P} = \mathbf{P}^H$ holds for all orthogonal projection matrices \mathbf{P} .

Step 3: The DC estimate is derived by

$$\mathbf{Q}_T \underline{a} \hat{d} = \underline{x}''_a \quad (15)$$

which leads to

$$\hat{d} = \frac{\mathbf{a}^H \mathbf{Q}_T \underline{x}}{\mathbf{a}^H \mathbf{Q}_T \mathbf{a}}. \quad (16)$$

The estimated DC offset vector $\underline{a} \hat{d}$ is the projection of \underline{x}''_a to the line $\underline{a} \mathcal{C}$.

The channel impulse response is estimated by the following two projection steps.

Step 4: The estimated DC offset is subtracted from the received signal:

$$\underline{x}' = \underline{x} - \underline{a} \hat{d} \quad (17)$$

The result is a projection of \underline{x} to the subspace orthogonal to $\underline{a} \mathcal{C}$. The projection \underline{x}' is projected to the subspace $\mathbf{T} \mathcal{C}^{L+1}$:

$$\underline{x}'_T = \mathbf{P}_T \underline{x}' = \mathbf{T} (\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T (\underline{x} - \underline{a} \hat{d}) \quad (18)$$

The channel estimate is derived by

$$\mathbf{T} \hat{\mathbf{h}} = \underline{x}'_T \quad (19)$$

so that we obtain

$$\hat{\mathbf{h}} = (\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T (\underline{x} - \underline{a} \hat{d}) \quad (20)$$

for the channel estimate.

Please note that Equations (16) and (20) do not necessarily define an efficient channel estimation algorithm. Sub-optimal simplifications and approximations exist as well as different derivations. However, all linear joint DC and channel estimation algorithms based on training sequence symbols have similar sensitivity to additive white Gaussian noise and IQ imbalance. This sensitivity is described in the next section.

4. CHANNEL ESTIMATION ERRORS

DC estimation errors have impact on the channel estimation. Moreover, for the equalization the DC-compensated samples \underline{x}' according to Equation (17) are used. An error of the DC estimation directly biases \underline{x}' .

From Equations (9) and (16) as well as from $\mathbf{Q}_T \mathbf{T} = \mathbf{0}$ we derive the DC estimation error

$$\hat{d} - d = \eta \underbrace{\frac{\mathbf{a}^H \mathbf{Q}_T \mathbf{A}^2 \mathbf{T} \mathbf{h}^*}{\mathbf{a}^H \mathbf{Q}_T \mathbf{a}}}_{e_{\hat{d},n}} + \underbrace{\frac{\mathbf{a}^H \mathbf{Q}_T \mathbf{n}}{\mathbf{a}^H \mathbf{Q}_T \mathbf{a}}}_{e_{\hat{d},n}}. \quad (21)$$

First we look at the DC estimation error due to noise $e_{\hat{d},n}$. The energy of this error term is given by

$$E_{\hat{d},n} = \mathbb{E} \|e_{\hat{d},n}\|^2 = \frac{N_0}{\|\mathbf{Q}_T \mathbf{a}\|^2} \quad (22)$$

Obviously, the closer the vector \underline{a} to the subspace $\mathbf{T} \mathcal{C}^{L+1}$ is, the smaller $\|\mathbf{Q}_T \mathbf{a}\|$ and the larger $E_{\hat{d},n}$ is. The angles between \underline{a} and $\mathbf{T} \mathcal{C}^{L+1}$ for different training sequence codes ("tsc"s) are listed in Table 1.

We see that for GMSK (where the rotation angle equals $\phi = \pi/2 = 90^\circ$) the subspace angle is more than 55° for all training sequences. It goes up to 81° . However, for 8PSK (where the rotation angle equals $\phi = 3\pi/8 = 67.5^\circ$) and $L = 4$ the subspace angle is nearly 60° for training sequences number 0 and 1, and only roughly 40° for all other training sequences. The subspace angle is even less for larger L .

Table 1. Angle between \underline{a} and $\mathbf{T} \mathcal{C}^{L+1}$

tsc	0	1	2	3	4	5	6	7
L	GMSK							
4	65°	61°	65°	61°	79°	62°	81°	70°
5	61°	59°	61°	59°	77°	61°	79°	68°
6	59°	57°	59°	57°	73°	61°	77°	66°
L	8PSK							
4	57°	57°	40°	39°	39°	39°	39°	37°
5	51°	52°	35°	35°	33°	34°	34°	32°
6	44°	45°	28°	29°	27°	27°	27°	26°

Table 2. Noise suppression [dB] of the channel estimator

tsc	0	1	2	3	4	5	6	7
L	GMSK							
4	19.7	18.4	19.7	18.4	27.4	19.3	29.3	22.2
5	18.0	17.6	18.0	17.6	25.6	18.9	27.4	21.2
6	17.3	16.8	17.3	16.8	23.2	18.8	25.3	20.1
L	8PSK							
4	16.4	16.4	12.4	12.4	11.5	11.8	11.6	11.1
5	14.2	14.9	10.6	10.7	9.5	10.1	9.7	9.3
6	12.0	12.4	8.2	8.5	7.2	7.6	7.3	7.4

Table 3. Minimum imbalance suppression [dB] of the channel estimator

tsc	0	1	2	3	4	5	6	7
L	GMSK							
4	11.4	10.1	11.4	10.1	25.3	19.1	28.0	14.7
5	8.6	8.3	8.6	8.3	23.3	19.2	25.2	13.0
6	7.7	7.5	7.7	7.5	20.1	16.8	21.1	11.6
L	8PSK							
4	8.8	8.8	1.7	1.7	1.8	2.2	1.1	1.4
5	4.3	4.7	0.8	0.6	-1.3	-0.3	-1.2	-0.4
6	1.6	1.4	0.4	0.2	-3.9	-3.1	-3.7	-1.2

The different subspace angles are reflected in the channel estimation errors. The error due to noise is given by

$$\underline{e}_{\hat{h},n} = (\mathbf{T}\mathbf{T}^T)^{-1}\mathbf{T}^T \underline{a} e_{d,n}. \quad (23)$$

The noise suppression values $N_0/\|\underline{e}_{\hat{h},n}\|^2$ for different training sequence codes and channel orders are presented in Table 2.

The error due to IQ imbalance is given by

$$\underline{e}_{\hat{h},\eta} = (\mathbf{T}\mathbf{T}^T)^{-1}\mathbf{T}^T \underline{a} e_{d,\eta}. \quad (24)$$

The imbalance suppression $|\eta|^2\|\hat{h}\|^2/\|\underline{e}_{\hat{h},\eta}\|^2$ is a function of \hat{h} but the minimum value can be calculated using the Schwarz inequality. The minimum values for different training sequence codes and channel orders are presented in Table 3.

We can see that the noise suppression is excellent for GMSK (more than 17 dB). However, for 8PSK the noise suppression for training sequences number 2–7 is roughly 4 dB weaker than for training sequences number 0 and 1. The IQ imbalance suppression varies strongly for GMSK from 7.5 dB to 28 dB. For 8PSK, the IQ imbalance suppression even goes down to negative values.

Besides these two error terms, IQ imbalance and noise add errors to the channel estimates even in case of an error-free DC estimator. However, these errors are negligible with respect to our conclusions on the training sequence dependency of zero IF EDGE receivers.

5. CONCLUSION

We defined an optimum linear channel estimator for GSM/EDGE direct conversion receivers. We derived estimation errors of this estimator caused by noise and IQ gain/phase imbalance. We have shown that the selection of the training sequence has a significant influence on these error terms.

A symbol-by-symbol rotation $\phi = \pi/2$ is better than rotation by $\phi = 3\pi/8$. This is very unfortunate, because $\phi = \pi/2$ is used for GMSK which is less sensitive to estimation errors and a joint DC and channel estimation can be replaced by a pure channel estimation (neglecting the residual DC of blind estimation procedures prior to channel estimation) so that the derived training sequence dependent errors do not occur at all. The choice of $\phi = 3\pi/8$ for 8PSK leads to even more sensitivity to distortions of 8PSK modulation and coding schemes.

For 8PSK training sequences number 0 and 1 lead to better detection results than other training sequences, especially number 5 to 8. This means that bit/block error rate simulations and measurements with bursts using one training sequence do not represent simulations and measurements with bursts using other sequences. EDGE receiver simulation results have to be questioned even more critically.

6. REFERENCES

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