

On Motion Models for Target Tracking in Automotive Applications

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Abstract— In automotive tracking applications, using two separate linear state space models for longitudinal and angular movement of objects is a widely applied simplification. The separation is possible if the observed targets are positioned straight ahead and moving in approximately the same direction as the observer, like in adaptive cruise control (ACC) systems. However, in more general scenarios of future tracking applications, object motion may not be limited to certain directions. In inner-city and intersection situations, other road users are passing even perpendicular to the observing vehicle. Most tracking systems of today are not prepared to handle those situations, as the simplified modeling is no longer appropriate.

In the paper on hand we will review the commonly used models and state their main drawbacks. The conclusion of these drawbacks is the use of a motion model which reflects a more natural description of typical objects to be considered in automotive applications. All mathematical expressions necessary for an implementation using an extended/unscented Kalman filter are provided.

The state space model was designed for radar target tracking but is not limited to radar. With modifications to the measurement equations, the model can be used for camera-based systems as well as for ultrasonic sensors or laserscanner systems.

Index Terms—Radar tracking, Radar signal processing, Road vehicle radar

I. INTRODUCTION

Many of today's automotive driver assistance systems are focusing on the area in front of the observing vehicle. More specific, objects to be considered as dangerous or relevant for speed adaptation are expected to be positioned in the own predicted way of travel. The sensors to monitor this region are mounted in the front of the vehicle and are directed to the front (Fig. 1 shows the principal fields of view of three sensors as an example).

Certain driver assistance systems are designed to react in situations where the observing vehicle is following other vehicles in the same or nearly the same direction. With these limitations, movements of an object in longitudinal direction relative to the observing vehicle (in x -direction in Fig. 1) are caused by acceleration/deceleration of one vehicle, either object or observer. On the other hand, movements in transversal/angular direction occur if one of both vehicles changes its driving direction. With these facts in mind, the two different types of relative object motion can be separated. In the tracking implementation, this separation is reflected by the use of two separate linear Kalman filters, one for longitudinal and one for transversal movement [1] [2]. The filter parameters, specifically the input variances of the Kalman filters, can be derived by considering the average acceleration/deceleration and the average change in driving direction (gyro rate) that are likely to occur in typical road traffic scenarios.

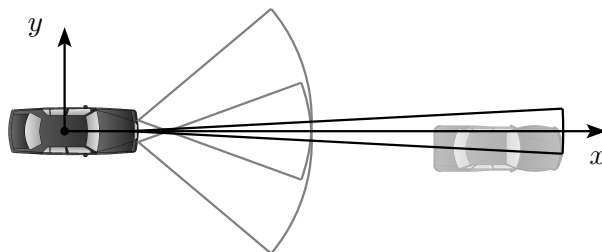


Fig. 1. Typical mounting positions and fields of view

In more advanced applications such as blind spot surveillance and driver assistance in inner-city situations, there is the need for an extension of the sensor's field of view all around the vehicle. Clearly, the limitations of the object movement mentioned above are no longer acceptable. Even objects that are moving perpendicular to the own driving direction have to be considered. In that case, transversal movement is no longer solely caused by a change in driving direction of object or observer. The two types of influence of a

driver on the vehicle movement (turning the steering wheel and accelerating/braking) are no longer separable in the same way.

Due to this fact, we propose using a different motion model with a more general description of object movement. In this model, movement is no longer modeled in terms of longitudinal/transversal motion relative to the sensor coordinate system. Instead, the tangential speed and the heading angle/driving direction are represented by state variables. The model equations are simplified by using a fixed (global) coordinate system where both the observing vehicle and the observed objects are moving through. Clearly, the state and measurement equations are nonlinear. All equations and expressions necessary for the implementation of an extended or unscented Kalman filter are given in the appendix.

In the next section the commonly used state space model and its main drawbacks are described in detail. The proposed state space model is introduced in section III; additional comments on the choice of the global coordinate system are given in section IV. After some words about the problem of track initialization in section V, a comparison between the performance of both models is given by simulation results in section VI.

II. COMMON STATE SPACE MODEL

A. Model equations

The separated state space model referred to in the introduction is presented in [1]. In contrast to the cited paper, here we will state all variables in discrete time with time index k .

In a simple model for the longitudinal motion, one would include the distance to the object, the relative speed between object and observer and, optionally, the relative acceleration. Using the inertial measurements of the own velocity as additional information about the motion of the own vehicle, this model can be enhanced. The model for the longitudinal movement as in [1] then includes the following state variables: The distance between observer and object $d(k)$, the speeds of the observer $v_{\text{ego}}(k)$ and object $v_{\text{obj}}(k)$ as well as the corresponding accelerations $a_{\text{ego}}(k)$ and $a_{\text{obj}}(k)$. Summarizing, the motion can be described by the state transition equation

$$\mathbf{x}(k+1) = \begin{bmatrix} d(k+1) \\ v_{\text{ego}}(k+1) \\ v_{\text{obj}}(k+1) \\ a_{\text{ego}}(k+1) \\ a_{\text{obj}}(k+1) \end{bmatrix} = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{w}(k) \quad (1)$$

with state transition matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -T & T & -\frac{T^2}{2} & \frac{T^2}{2} \\ 0 & 1 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

and cycle time T . Changes in the accelerations of observer and object are modeled by the 2×1 -vector of noise processes $\mathbf{w}(k)$, whose elements are mapped to the last two state variables by the matrix

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T. \quad (3)$$

The state transition equations for the angular dynamics model (index a) has the following form:

$$\mathbf{x}_a(k+1) = \begin{bmatrix} \alpha(k+1) \\ \dot{\alpha}(k+1) \end{bmatrix} = \mathbf{x}_a(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_a(k) \quad (4)$$

Here, $\alpha(k)$ is the relative angle from the vehicle (or sensor) axis to the object and $\dot{\alpha}(k)$ is the angular velocity or gyro. Changes of the angular velocity over time are modeled by the input noise process $w_a(k)$.

While the state transition equation describes the motion of an object in general, the measurement equation depends on the sensor in use. Like in our case, in [1] the tracking system is designed for automotive radar sensors. These measure the distance and relative speed to objects inside their field of view. By additionally using the inertial speed measurement $v_{\text{ego}}^{\text{m}}(k)$, the following measurement equation for the longitudinal model results:

$$\mathbf{y}(k) = \begin{bmatrix} d^{\text{m}}(k) \\ v_{\text{rel}}^{\text{m}}(k) \\ v_{\text{ego}}^{\text{m}}(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \mathbf{v}(k). \quad (5)$$

Measurements are marked by the superscript ^m; otherwise, they could be mixed with the corresponding state variables. In [1], measurements of the own acceleration are additionally used. But as these are not actually measurements but derived from the inertial speed measurements, we do not use them here. The vector $\mathbf{v}(k)$ represents white Gaussian measurement noise.

By using different beams and applying the mono-pulse/sequential lobing principle [3], automotive radar sensors are able to estimate the relative angle to the observed object. Using this information, the measurement equation for the angular model (4) is

$$y_a(k) = \alpha^{\text{m}}(k) = [1 \quad 0] \mathbf{x}_a(k) + v_a(k) \quad (6)$$

with the angle measurement noise process $v_a(k)$.

B. Model Drawbacks

As stated above, the given system model can be expected to work well under the limitation that the observed object is moving in nearly the same direction as the observing vehicle. In the future applications, as stated above, this limitation will no longer be acceptable.

As an example, we will have a look at a situation where an object passes with a constant distance of 10 m in x -direction *perpendicular* to the observing vehicle with a speed of 30 km/h. The resulting distance and relative angle are shown in Fig. 2. Both distance and relative angle are changing in a nonlinear way, even if the observed object has a constant motion state. These nonlinear changes were artificially introduced by the definition of the state space model and mean additional work for the tracking filters.

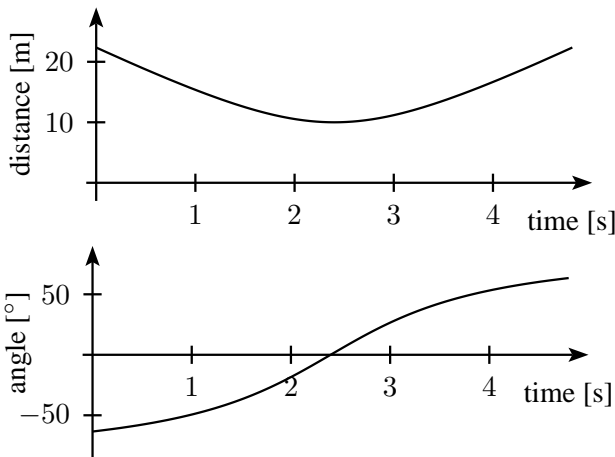


Fig. 2. Vehicle driving perpendicular to observer

Next, we will examine the measurement equation for the relative speed, i.e. the second row of equation (5),

$$v_{\text{rel}}(k) = v_{\text{obj}}(k) - v_{\text{ego}}(k). \quad (7)$$

This equation is only true or approximately fulfilled for the case of nearly equal driving directions, as was assumed during the design of the given model. In general, the relative speed between sensor and object, which can be measured, for example, with radar sensors, depends on the angle between the driving directions. The ego motion information should definitely be considered in the model to improve the tracking system, but in a different way. Only thanks to the well-known robustness of the Kalman filter, the tracking might still work even with this erroneous state space model.

In the commonly used, separated motion model, the state of an object is defined in sensor coordinates,

as it can easily be seen in (5) and (6). However, applications that use the object information as an input need the object position and speed in the vehicle coordinate system. Thus, a transformation from sensor to vehicle coordinate system is necessary. If more than one sensor is used, different nonlinear transformations from different sensor coordinate systems to vehicle coordinates are necessary. While the positions and speeds can easily be transformed, this is not the case for the state variances used in the Kalman filter. This is because a Gaussian distribution – all error distributions are usually assumed to be Gaussian in Kalman filtering – is no longer a Gaussian distribution after a nonlinear transformation. As the data of different sensors should be fused to a single object list before forwarding to the application, this fact makes the processing more complicated and error-prone.

III. A MORE GENERAL STATE SPACE MODEL

We can summarize the discussion of the common motion model in the last section as four requests for a more generally applicable motion model:

1. The two distinct types of movement changes (acceleration/deceleration and steering) shall be well separated.
2. Constant object motion shall be represented by a constant motion state in the model.
3. Ego- and object motion information have to be considered in a correct way.
4. All sensors should be working in the same coordinate system.

Fulfilling the first two requests is possible by describing the movement of an object by its absolute (i.e. tangential) speed and its current heading direction. The resulting state transition equation is the following:

$$\mathbf{x}(k+1) = \begin{bmatrix} s_x(k+1) \\ s_y(k+1) \\ v(k+1) \\ \varphi(k+1) \\ a(k+1) \\ \delta(k+1) \end{bmatrix} = \mathbf{f}(\mathbf{x}(k)) + \mathbf{B}\mathbf{w}(k). \quad (8)$$

The state vector $\mathbf{x}(k)$ consists of the position $(s_x(k), s_y(k))$, the tangential speed $v(k)$, the heading angle $\varphi(k)$ and the acceleration $a(k)$. The variable $\delta(k)$ represents the steering wheel angle and is used to model changes in the driving direction. Again, a 2×1 -vector of input noise processes $\mathbf{w}(k)$ is used to model changes in acceleration and steering wheel angle.

As stated in the introduction for the separated model, also here the parameters can be chosen based on the typical values of acceleration/deceleration and changes in driving direction. But here, in contrast, the representation is more general and not limited to a special case.

Obviously the transition function $f(\mathbf{x}(k))$ is nonlinear. The computational effort is thus larger compared to the common motion model. But the correct consideration of the measured ego- and relative speed is not possible using only linear equations. Function $f(\mathbf{x}(k))$ is derived in appendix A.

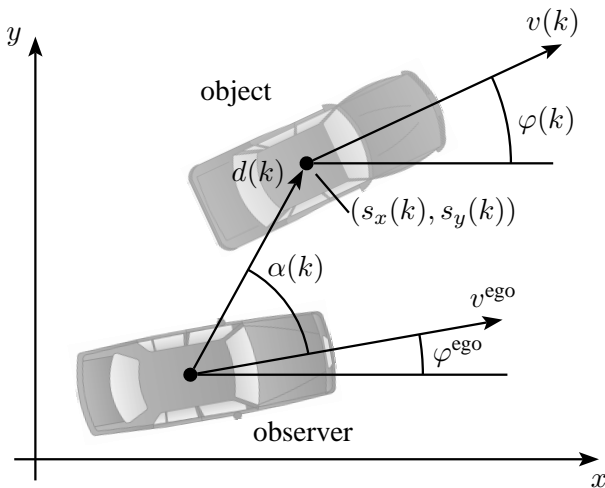


Fig. 3. State and measurement variables

Fig. 3 shows the main state variables used to model the motion of observer (superscript ^{ego}) and object. For sake of simplicity, the relative angle $\alpha(k)$ and distance $d(k)$ are sketched as if the sensor were in the center of the observer and the object were a point target.

The sensors under consideration measure distance, relative angle and Doppler speed. The resulting measurement equation is thus

$$\mathbf{y}(k) = \begin{bmatrix} d^m(k) \\ \alpha^m(k) \\ v_{\text{rel}}^m(k) \end{bmatrix} = \mathbf{g}(\mathbf{x}(k)) + \mathbf{v}(k), \quad (9)$$

and is as well nonlinear. Function $\mathbf{g}(\mathbf{x}(k))$ is given in appendix A. The 3×1 -vector $\mathbf{v}(k)$ is used to model the measurement noise in all three dimensions.

The choice of the state variables allows a 1-to-1 mapping of what we might call a constant motion state of a vehicle in colloquial words to a constant state in the motion model. If the driver keeps the steering wheel and the accelerator pedal fixed, constant acceleration $a(k)$ and constant steering wheel angle $\delta(k)$ will result.

As the steering wheel angle was chosen as a state variable, a model that relates the steering wheel angle to the driving direction is needed. A simple but sufficiently accurate model is the two-point bicycle model, like it is also used in [4] and [5]. The state transition equation for the driving direction results in

$$\varphi(k+1) = \varphi(k) + \frac{T}{L} \delta(k) v(k) + w_2(k). \quad (10)$$

Note that the wheel base L , i.e. the distance between front and rear wheels of the imaginary bicycle, is needed in this equation. Clearly, the true wheel base of an object observed by the sensor will not be available. But as using a fixed default value for the wheel base of all objects will only result in a scaled steering wheel angle, the model is generally applicable for tracking purposes.

The two-point bicycle model provides the advantage that it inherently relates changes in driving direction to the object speed. This avoids random changes in the driving direction of slowly moving objects due to measurement noise. As tracked objects can be assumed to be either some sort of road vehicle or fixed objects, the steering angle can be limited to a maximum value. Changes in the moving direction of vehicles (driving forwards or backwards) are then represented by a change of the sign of the tangential speed.

The coordinate system in which the object position $(s_x(k), s_y(k))$ is measured was not specified yet. One possible choice is to use the inertial coordinate system of the observer. This would fulfill request 4 stated before, but still the positions of objects would have to be transformed in every cycle according to position changes of the observing vehicle. To avoid this, we propose to use a fixed (or global) coordinate system. At system startup, the origin of this coordinate system can be defined arbitrarily, for example as the current position of the observer. The own vehicle motion is modeled using the same set of state variables and the same state transition equation (8), but with the inertial measurements of speed and steering angle instead of the radar measurements. As both observed objects and observing vehicle are moving through the same fixed coordinate system, the ego motion is considered correctly and the state transition equation is greatly facilitated.

IV. GLOBAL COORDINATES WITHOUT GPS?

A possible point of critics to our proposed model is the choice of a global coordinate system. The observing vehicle is moving through this coordinate system

and thus has to keep track of its own position. Without using a system delivering a global position estimation, like GPS, there is no other possibility than to integrate the measurements of the inertial sensors (speed and steering angle or gyro rate) over time (also called “dead reckoning”). Unfortunately, as the sensors are subject to noise (especially the gyro sensor), the error in the own position will be constantly growing. The question arises if this means that the proposed motion model mandatory requires a GPS system.

The answer to this question is no. Of course the error in the estimated vehicle position will soon reach dimensions where it is everything else than neglectable. But as the measurements of the target positions (using radar, ultrasonic, laser scanner or video) are still relative to the observing vehicle, the error in the position of the observed objects will be exactly the same as the error in the observer position. This means that the position errors will just cancel out each other. The global position $(s_x(k), s_y(k))$ itself is not of any help, but using the global coordinate system facilitates the motion model equations.

V. TRACK INITIALIZATION

The initialization of tracks is a special problem when objects moving in all possible directions have to be considered. While a single measurement vector of a radar sensor (measuring distance, bearing and Doppler speed) includes information about the object position, it includes only partial information about the movement – if no assumptions about the object motion, like that it is moving into the same direction as the observer, are made. Movement direction and speed can not be initialized reliably with a single measurement. Due to measurement noise and severe quantization in certain sensor types, even two or three radar measurements may provide no or misleading information about the object motion. Thus, a special treatment is necessary for the initialization of tracks. Several measurements have to be collected before the Kalman filter can be started.

The initialization problem is not specific for the proposed model, but naturally arising when objects moving in all possible directions are to be tracked. However, as the topic of this paper is the state space model and not the whole tracking process, we will not provide any more details here.

VI. SIMULATION RESULTS

In order to be able to compare the performance of both models objectively, data with an exact reference

is needed. Here, we use radar data that was generated by our own radar target list simulation which delivers very realistic radar data [6] [7]. We have chosen a scenario in which a point target moves with a speed of about 10 km/h on a circle of 15 m radius through the view field of a front-mounted short-range radar.

Both state space models discussed before were used to track the simulated targets. The tracking algorithm with the common motion model was implemented as described in [1]. For the new motion model, we used an extended Kalman filter. Due to the mentioned problems in the track initialization, the known true state was used as initialization. The eu-

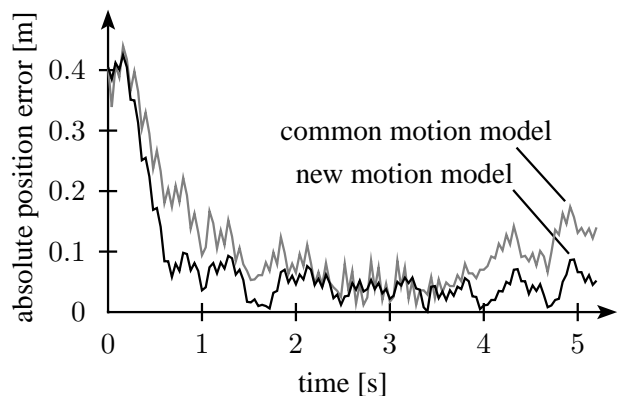


Fig. 4. Absolute position error of circular moving object

clidean distance between the estimated track position and the true object position is shown for both models in Fig. 4. Note that the comparably large error at the beginning is due to large measurement errors at the edge of the radar sensor’s field of view. The figure shows that the position error with the new motion model is lower over nearly the whole track lifetime.

VII. CONCLUSION

We have presented a motion model for tracking in automotive radar applications that is superior to commonly used linearized models. A global coordinate system allows the simple fusion of data of different sensors mounted on one vehicle. Further, by using the tangential vehicle speed and the driving direction as state variables, the two types of movement changes (acceleration/deceleration and change of driving direction) of vehicles are well separated and allow a natural description of object movement. Simulation results show that the new motion model outperforms the commonly used approach.

APPENDIX

A. Model Functions

In this section, the state transition and measurement functions according to the introduced motion model are derived.

For the derivation of the state transition function $f(\mathbf{x}(k))$, we use a continuous-time representation of some variables (denoted by index c) for the moment. With this, the exact equation for the position variables s_x and s_y at time index $k + 1$ or time $t = (k + 1)T$ is

$$\begin{bmatrix} s_x(k+1) \\ s_y(k+1) \end{bmatrix} = \begin{bmatrix} s_x(k) \\ s_y(k) \end{bmatrix} + \int_{kT}^{(k+1)T} v_c(t) \begin{bmatrix} \cos(\varphi_c(t)) \\ \sin(\varphi_c(t)) \end{bmatrix} dt \quad (11)$$

with the time-continuous speed $v_c(t)$ and driving direction $\varphi_c(t)$. As all estimations are later done from cycle to cycle, it is straightforward to assume that speed and driving direction are linearly changing in an interval of length T . But this assumption leads to complicated and numerically unfavorable state transition equations with two different expressions for the cases where the steering angle is equal or not equal to zero. Due to this, we approximate speed and driving direction as constant during one time interval of length T and set them to the values that would result at the interval center $t = (k + \frac{1}{2})T$ in the linear case:

$$v_c(t) = \bar{v}(k) = v(k) + \frac{T}{2} a(k) \quad (12)$$

$$\varphi_c(t) = \bar{\varphi}(k) = \varphi(k) + \frac{T}{2L} \delta(k) v(k). \quad (13)$$

The following simplified transition equations results:

$$\begin{bmatrix} s_x(k+1) \\ s_y(k+1) \end{bmatrix} = \begin{bmatrix} s_x(k) \\ s_y(k) \end{bmatrix} + T\bar{v}(k) \begin{bmatrix} \cos(\bar{\varphi}(k)) \\ \sin(\bar{\varphi}(k)) \end{bmatrix} \quad (14)$$

Following our experience of numerous test runs, the given approximations do not cause any noticeable errors. The transition equations for the remaining state variables speed, acceleration and steering angle are linear:

$$\begin{bmatrix} v(k+1) \\ a(k+1) \\ \delta(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v(k) \\ a(k) \\ \delta(k) \end{bmatrix}. \quad (15)$$

Together, equations (10), (14) and (15) form the state transition function $f(\mathbf{x}(k))$ of equation (8).

The measurement equations can be derived with help of Fig. 3. Let $(s_x^{\text{sen}}, s_y^{\text{sen}})$ be the current sensor position in the global coordinate system, φ^{ego} the current driving direction of the observer and ϑ^{sen} the look direction of the sensor with respect to the longitudinal

vehicle axis. The following equations can then easily be derived:

$$\alpha(k) = \arctan\left(\frac{s_y(k) - s_y^{\text{sen}}}{s_x(k) - s_x^{\text{sen}}}\right) - \varphi^{\text{ego}} - \vartheta^{\text{sen}} \quad (16)$$

$$d(k) = \sqrt{(s_x(k) - s_x^{\text{sen}})^2 + (s_y(k) - s_y^{\text{sen}})^2} \quad (17)$$

$$\begin{aligned} v_{\text{rel}}(k) &= v^{\text{ego}} \cos(\alpha(k) + \vartheta^{\text{sen}}) \\ &\quad - v(k) \cos(\alpha(k) + \vartheta^{\text{sen}} + \varphi^{\text{ego}} - \varphi(k)) \end{aligned} \quad (18)$$

Equations (16), (17) and (18) represent the measurement function $g(\mathbf{x}(k))$ of the proposed motion model in equation (9).

As stated before, the error in the estimated global position will grow over time. This is reflected by growing variances of the corresponding states in the Kalman filter. However, these variances are never used (only after multiplication with zero) and can thus be set to an arbitrary and fixed value to avoid overflow problems.

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