# A GLOBAL MOTION MODEL FOR TARGET TRACKING IN AUTOMOTIVE APPLICATIONS

*Markus Bühren and Bin Yang*

Chair of System Theory and Signal Processing
University of Stuttgart, Germany
*www.LSS.uni-stuttgart.de*

**ABSTRACT**

In common automotive radar tracking systems, simple linear models are used to track targets separately in longitudinal and angular direction relative to the own vehicle (or sensor) position. Under the special condition that the observed targets are straight ahead and moving nearly in the same direction as the observing vehicle, like in adaptive cruise control (ACC) systems, those models work well. In more general scenarios, where movements of other vehicles have to be tracked in all possible directions and all around the vehicle (e.g. in inner-city or intersection situations), the modeling is insufficient.

In this paper we review the drawbacks of the commonly used models and present a more general motion model for automotive tracking systems. All necessary expressions for an implementation using an extended or unscented Kalman filter are given. Even if designed for radar systems, the state model is not limited to a special type of sensor. It can be used for ultrasonic or laser scanner systems as well as for vision-based systems with a different measurement model.

***Index Terms*—** Radar tracking, Radar signal processing, Road vehicle radar

## 1. INTRODUCTION

The automotive driver assistance systems of today often focus on the area in front of the vehicle, especially on the predicted way of travel. The sensors are mounted in the front of the vehicle and are directed to the front so that their field of view is in the driving direction (Fig. 1 shows a typical constellation with one far- and two near-range sensors). Further, those driver assistance systems are designed to react in situations where the observing vehicle is following other vehicles in the same or nearly the same direction. Under these conditions, a longitudinal object movement relative to the observing vehicle (i.e. mainly in x-direction, Fig. 1) is caused by an acceleration or deceleration of either the observing vehicle or the observed vehicle. A relative movement in angular or transversal direction, on the other hand, is caused by a change of the steering wheel angle of either vehicle. These facts allow a computationally very simple model by separating the two types of movement (acceleration/deceleration and steering). They are mapped on two separate linear Kalman filters for the longitudinal and angular directions [1] [2]. The input variances needed for the Kalman filter can be derived by examining the maximum (or average) expected acceleration/deceleration for the longitudinal dynamics and by finding the maximum (or average) expected change in driving direction (gyro rate) of a vehicle for the angular dynamics.

Future applications of automotive radar will entail the need to extend the field of view to other regions around the vehicle, e.g. for blind spot surveillance and lane change assistance. Additional assistance functions for special traffic situations like intersections are already under consideration. This motivates tracking systems that are not limited to objects that are in front of the observing vehicle and move in the same direction, but also objects that are, for example, moving perpendicular to the own driving direction. In this case, a transversal movement (i.e. a change in the relative angle to the object) is no longer solely caused by turning the steering wheel. While in the restricted situation stated above the two forms of movement are well separated, this is clearly not the case in general. For this reason, we propose to use a different motion model that describes object movements not by longitudinal and angular speeds (and accelerations, respectively) relative to the sensor, but by the tangential speed and the heading angle/driving direction. This model formulation is simplified by using a global (i.e. fixed) coordinate system where both the observing vehicle and the observed objects are moving through. The state and measurement equations of this model are clearly nonlinear. We will present the equations and expressions necessary for an implementation with an extended or unscented Kalman filter.

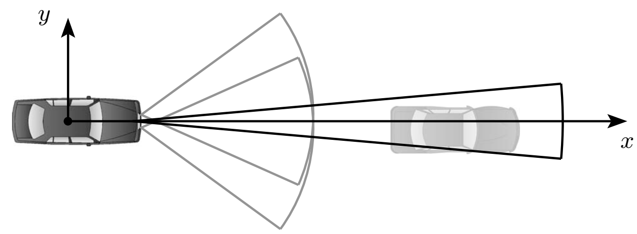


**Fig. 1**. Typical mounting positions and fields of view

In section 2 the commonly used state space model and its main drawbacks are described in detail. The new state space model is presented in section 3. Further discussion of the choice of the global coordinate system is done in section 4. Finally, in section 5 simulation results are presented in order to compare the two models and to show the ability of the proposed model to track objects all around the observing vehicle.

## 2. COMMON STATE SPACE MODEL

### 2.1. Model equations

The common state space model that was mentioned in the introduction is presented in [1]. In contrast to the cited paper, we will state all variables and equations in discrete time with time index $k$. In the longitudinal direction, a simple state space model would include the distance between own vehicle and object, the relative speed and the relative acceleration. But as in most vehicles a measurement of the own velocity is available (as it is necessary for the ESP system), this state model can be enhanced by taking this additional information into account. In [1], the final model for the longitudinal dynamics consists of the following state variables: Distance to the object $d(k)$, inertial speeds of the own vehicle $v_{\text{ego}}(k)$ and the object $v_{\text{obj}}(k)$ and the corresponding accelerations $a_{\text{ego}}(k)$ and $a_{\text{obj}}(k)$. With these states, the state transition equation

$$\mathbf{x}(k+1) = \begin{bmatrix} d(k+1) \\ v_{\text{ego}}(k+1) \\ v_{\text{obj}}(k+1) \\ a_{\text{ego}}(k+1) \\ a_{\text{obj}}(k+1) \end{bmatrix} = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{w}(k) \tag{1}$$

with the state transition matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -T & T & -\frac{T^2}{2} & \frac{T^2}{2} \\ 0 & 1 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and the cycle time $T$ results. Here, $\mathbf{w}(k)$ is a $2\times1$-vector of noise processes that model the change of the accelerations of the observing vehicle and the object, and $\mathbf{B}$ is the noise input matrix that maps the elements of $\mathbf{w}(k)$ to the last two state variables.

The separate model for the angular dynamics (index $a$) looks as follows:

$$\mathbf{x}_a(k+1) = \begin{bmatrix} \alpha(k+1) \\ \dot{\alpha}(k+1) \end{bmatrix} = \mathbf{x}_a(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_a(k) \tag{3}$$

Here, $\alpha(k)$ is the relative angle from the vehicle (or sensor) axis to the object and $\dot{\alpha}(k)$ is the angular velocity (often referred to as gyro). The input noise process $w_a(k)$ models the change of the angular velocity over time.

The measurement equation depends on the sensor in use. In [1], the tracking system is designed for the use with automotive radar sensors. These sensors typically deliver measurements of distance and relative speed to targets inside their field of view. Using the inertial speed measurement $v^{\text{m}}_{\text{ego}}(k)$, the resulting measurement equation for the longitudinal model is thus

$$\mathbf{y}(k) = \begin{bmatrix} d^{\text{m}}(k) \\ v^{\text{m}}_{\text{rel}}(k) \\ v^{\text{m}}_{\text{ego}}(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \mathbf{v}(k). \tag{4}$$

The superscript $^{\text{m}}$ helps to distinguish between the state variables and the corresponding measurements. In comparison to [1], we ignore the measurement of the own acceleration because it is not actually measured but derived from the inertial speed measurements. The vector $\mathbf{v}(k)$ consists of white Gaussian random variables representing the measurement noise.

Further, by using different beams and applying the monopulse/sequential lobing principle [3], the relative angle to the observed object is estimated. With this, the measurement equation for the angular model (3) is

$$y_a(k) = \alpha^{\text{m}}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x_a}(k) + v_a(k) \tag{5}$$

with the angle measurement noise process $v_a(k)$.

### 2.2. Drawbacks

As stated above, the given system model is expected to serve well for the case that the observed object is moving in nearly the same direction as the observing vehicle. In the future, however, the need for tracking systems that are able to cover more general object movements, as they naturally occur, for example, in inner-city scenarios, will arise. Here, the assumption of equal driving directions is not motivated any more.

Let us have a look at a situation where an object is driving in a constant distance of 10 m in $x$-direction *perpendicular* to the driving direction of the observing vehicle with a speed of 30 km/h. The resulting distance and relative angle are shown in Fig. 2. As is clearly observable, both distance and relative angle change in a nonlinear way, even if the observed object has constant speed and constant driving direction. These nonlinear changes are not physically given but were artificially introduced by the definition of the state space model.

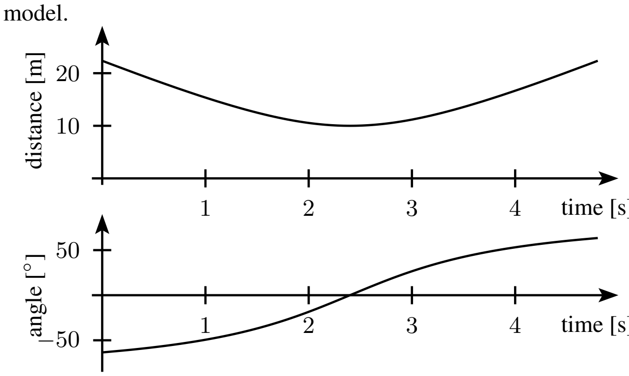


**Fig. 2**. Vehicle driving perpendicular to observer

We now focus on the measurement equation for the relative speed, i.e. the second row of equation (4),

$$v_{\text{rel}}(k) = v_{\text{obj}}(k) - v_{\text{ego}}(k). \tag{6}$$

This equation is only true for the case of equal driving directions, as it was mentioned before. In general, the relative speed between observer (sensor) and object, which can be measured, for example, with radar sensors, depends on the angle between the driving directions. The ego motion information is definitely expected to improve the tracking system, but has to be considered in a different way. Only due to the well-known robustness of the Kalman filter, the tracking might still work even with this erroneous state space model.

In the common motion model, the state of an object is defined in sensor coordinates, as it can easily be derived from the measurement equations (4) and (5). Nearly every application that uses the object state as an input (e.g. the adaptive cruise control) needs the object position and speed in the vehicle coordinate system. Thus, a transformation from sensor to vehicle coordinate system is necessary. If more than one sensor is used, the transformations from the sensor coordinate systems to the vehicle coordinate system are different. The state variances can not simply be transformed to another coordinate system, as a Gaussian distribution (all error distributions are usually assumed to be Gaussian when using the Kalman filter) is no longer a Gaussian distribution after a nonlinear transformation. This makes the fusion of the objects of different sensors more complicated and error-prone.

## 3. A GLOBAL STATE SPACE MODEL

The discussion of the common motion model in the last section can be summarized as four requests for a more generally applicable motion model:

R1. The two types of movement changes shall be separated.

R2. Constant motion shall result in constant movement state.

R3. Ego- and object motion have to be considered correctly.

R4. Common coordinate system for all sensors.

One straightforward way to fulfill the requests R1 and R2 is to describe the movement of an object by its absolute (i.e. tangential) speed and its current heading direction. The resulting state transition equation is the following:

$$\mathbf{x}(k+1) = \begin{bmatrix} s_x(k+1) \\ s_y(k+1) \\ v(k+1) \\ \varphi(k+1) \\ a(k+1) \\ \delta(k+1) \end{bmatrix} = \mathrm{f}\left(\mathbf{x}(k)\right) + \mathbf{B}\mathbf{w}(k). \tag{7}$$

The state vector consists of the position $(s_x(k),\ s_y(k))$, the tangential speed $v(k)$, the heading angle $\varphi(k)$ and the acceleration $a(k)$. The variable $\delta(k)$ is the steering wheel angle and models changes in the driving direction. Changes in the motion state are modeled by the $2 \times 1$-input noise vector $\mathbf{w}(k)$ which consists of noise processes for the acceleration and the change in driving direction.

Clearly, the transition function $\mathrm{f}\left(\mathbf{x}(k)\right)$ is nonlinear, which is a drawback compared to the common motion model. But the correct consideration of the measured ego- and relative speed is not possible using only linear equations. The transition function $\mathrm{f}\left(\mathbf{x}(k)\right)$ is derived in appendix A.

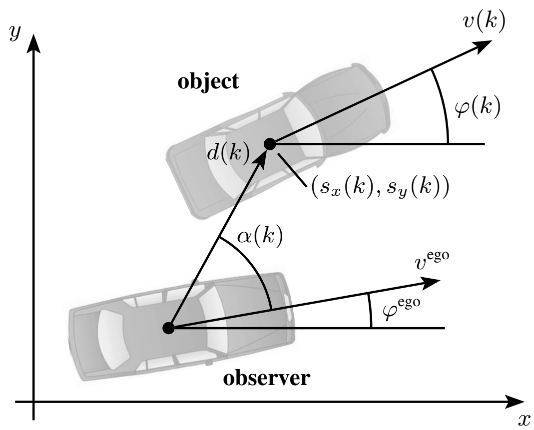


**Fig. 3**. State and measurement variables

Fig. 3 shows the main state variables for observer (superscript $^{\mathrm{ego}}$) and object vehicle. For sake of simplicity, the measured angle $\alpha(k)$ and distance $d(k)$ are sketched as if the sensor were in the center of the observer and the object were a point target.

The sensors we are considering here measure distance, relative angle and relative speed. The resulting measurement equation,

$$\mathbf{y}(k) = \begin{bmatrix} d^{\mathrm{m}}(k) \\ \alpha^{\mathrm{m}}(k) \\ v^{\mathrm{m}}_{\mathrm{rel}}(k) \end{bmatrix} = \mathrm{g}\left(\mathbf{x}(k)\right) + \mathbf{v}(k), \tag{8}$$

is nonlinear as well. The function $\mathrm{g}\left(\mathbf{x}(k)\right)$ is given in appendix A. The $3\times1$-vector $\mathbf{v}(k)$ models the measurement noise in all three dimensions.

The choice of the state variables allows a 1-to-1 mapping of what we could call a constant motion state of a vehicle in colloquial words to a constant state in the motion model. If the driver keeps the steering wheel and the accelerator pedal fixed, constant acceleration $a(k)$ and constant steering wheel angle $\delta(k)$ result.

As the steering wheel angle was chosen as a state variable, a model that relates the steering angle with the change in driving direction is needed. A simple and for tracking purposes sufficiently accurate model is the two-point bicycle model, as it is also used in [4] and [5]. The state transition equation of the driving direction results in

$$\varphi(k+1) = \varphi(k) + \frac{1}{L} \cdot T \cdot \delta(k) \cdot v(k) + w_2(k). \tag{9}$$

Note that the wheel base $L$, i.e. the distance between front and rear wheel of the imaginary bicycle, is needed in the model. Clearly, the true wheel base of an object observed by the sensor will never be available. But as using a fixed standard value for the wheel base for all objects will only result in a scaled steering angle, the model is generally applicable for tracking purposes. The two-point bicycle model has the advantage that it inherently relates the possible change in driving direction to the object speed. This avoids random changes in the driving direction of static objects. If a currently tracked object is known to be some sort of vehicle, the steering angle can be limited to a maximum value.

The coordinate system in which the object position $(s_x(k), s_y(k))$ is measured was not specified yet. One possible choice is to use the inertial coordinate system of the observer. In this case, the object position would have to be transformed in every step according to the new position of the observing vehicle. To avoid this, we have chosen to use a fixed (or global) coordinate system. When the tracking system is started, the origin of this coordinate system can be defined arbitrarily, for example as the starting position of the observer. The own vehicle motion is modeled using the same set of state variables, but with different measurements (speed and gyro rate/steering angle). As both the observed objects and the observing vehicle itself are moving through the same fixed coordinate system, the ego motion is considered correctly and the state transition equation is greatly facilitated.

## 4. GLOBAL COORDINATES WITHOUT GPS?

One argument against our proposed model might be the choice of a global coordinate system. As the observing vehicle is moving through this coordinate system, it has to maintain its own position. Without using a system which delivers a global position estimation, like GPS, the only way to keep track of the own position are the inertial speed and gyro rate/steering angle sensors. As the current vehicle position is computed by integrating the measurements (sometimes called "dead reckoning") and the sensors are corrupted by measurement noise (especially the gyro sensor), the position error will clearly rise over time. Is the conclusion that the proposed motion model will mandatory require a GPS system?

The answer to this question is no. Of course, after a long ride, the error in the estimated vehicle position will reach some hundred meters or even kilometers. But as the measurements of the sensors (radar, laser scanner or video) are still relative to the observing vehicle, the error in the position of the observed objects will be exactly the same as the error in the observer position. This means that the position errors will exactly cancel out each other. The global position $(s_x(k), s_y(k))$ itself is not of any help, but the choice of the global coordinate system facilitates the motion model equations.

## 5. SIMULATION RESULTS

In order to be able compare the performance of both models objectively, data with an exact reference is needed. Here, we use radar data that was generated by our own radar target list simulation which delivers very realistic radar data [6] [7]. We have chosen a scenario in which a point target moves with a speed of about 10km/h on a circle of 15m radius through the view field of a front-mounted short-range radar.

The two above-mentioned models were used to track the simulated radar target positions. The tracking algorithm with the common motion model was implemented as described in [1]. For the new motion model, we used an extended Kalman filter.

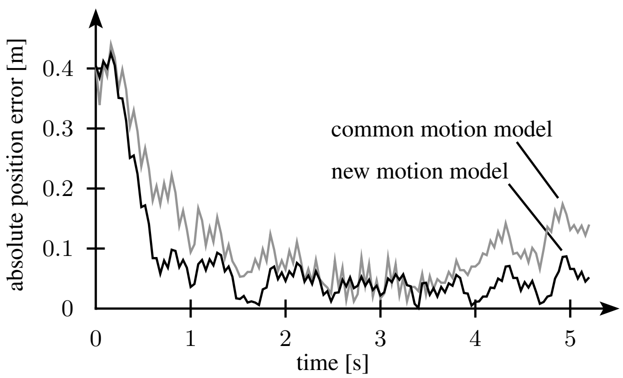


**Fig. 4**. Absolute position error of circular moving object

The euclidean distance between the estimated track position and the true object position is shown for both models in Fig. 4. Note that the comparably large error at the beginning is due to large measurement errors at the edge of the radar sensor's field of view. The figure shows that the position error with the new motion model is lower over nearly the whole track lifetime.

## 6. CONCLUSION

We have presented a motion model for tracking in automotive radar applications that is superior to commonly used linearized models. A global coordinate system allows the simple fusion of data of different sensors mounted on one vehicle. Further, by using the tangential vehicle speed and the driving direction as state variables, the two types of movement (acceleration/deceleration and steering angle) of vehicles are well separated and allow a natural description of object movement. Simulation results show that the new motion model outperforms the commonly used approach.

## A. MODEL FUNCTIONS

In this section, the state transition and measurement functions according to the introduced motion model are derived.

For the derivation of the state transition function $\mathrm{f}(\mathbf{x}(k))$, we use continuous-time versions of some variables (denoted by index $c$) for the moment. With this, the exact equation for the position variables $s_x$ and $s_y$ at time index $k+1$ or time $t = (k+1)T$ is

$$\begin{bmatrix} s_x(k+1) \\ s_y(k+1) \end{bmatrix} = \begin{bmatrix} s_x(k) \\ s_y(k) \end{bmatrix} + \int_{kT}^{(k+1)T} v_c(t) \begin{bmatrix} \cos(\varphi_c(t)) \\ \sin(\varphi_c(t)) \end{bmatrix} \mathrm{d}t \qquad (10)$$

with the time-continuous speed $v_c(t)$ and driving direction $\varphi_c(t)$. Speed and driving direction can be assumed to be linearly changing in an interval of length $T$. But as this assumption leads to complicated and numerically unfavorable state transition equations, approximate speed and driving direction as constant during one interval. They are set to the values that would result at the interval center $t = (k + \frac{1}{2})T$ using linear models:

$$v_c(t) = \bar{v}(k) = v(k) + \tfrac{T}{2} \cdot a(k) \qquad (11)$$

$$\varphi_c(t) = \bar{\varphi}(k) = \varphi(k) + \tfrac{T}{2} \cdot \tfrac{1}{L} \cdot \delta(k) \cdot v(k). \qquad (12)$$

The following simplified transition equations results:

$$\begin{bmatrix} s_x(k+1) \\ s_y(k+1) \end{bmatrix} = \begin{bmatrix} s_x(k) \\ s_y(k) \end{bmatrix} + T\bar{v}(k) \begin{bmatrix} \cos(\bar{\varphi}(k)) \\ \sin(\bar{\varphi}(k)) \end{bmatrix} \qquad (13)$$

Up to our experience, the given approximations do not cause any significant errors. The transition equations for the remaining state variables speed, acceleration and steering angle are straightforward and are omitted here. Together they form the state transition function $\mathrm{f}(\mathbf{x}(k))$ of equation (7).

The measurement equations can be derived with help of Fig. 3. Let $(s_x^{\text{sen}}, s_y^{\text{sen}})$ be the present sensor position in the global coordinate system, $\varphi^{\text{ego}}$ the current driving direction of the observer and $\vartheta^{\text{sen}}$ the look direction of the sensor with respect to the longitudinal vehicle axis. The following equations can then easily be derived:

$$\alpha(k) = \arctan\left(\frac{s_y(k) - s_y^{\text{sen}}}{s_x(k) - s_x^{\text{sen}}}\right) - \varphi^{\text{ego}} - \vartheta^{\text{sen}} \qquad (14)$$

$$d(k) = \sqrt{\left(s_x(k) - s_x^{\text{sen}}\right)^2 + \left(s_y(k) - s_y^{\text{sen}}\right)^2} \qquad (15)$$

$$\begin{aligned} v_{\text{rel}}(k) = {} & v^{\text{ego}} \cos\left(\alpha(k) + \vartheta^{\text{sen}}\right) \\ & - v(k) \cos\left(\alpha(k) + \vartheta^{\text{sen}} + \varphi^{\text{ego}} - \varphi(k)\right) \end{aligned} \qquad (16)$$

Together, equations (14), (15) and (16) form the measurement function $\mathrm{g}(\mathbf{x}(k))$ of the new motion model.

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