RECURSIVE ESTIMATION OF ROOM IMPULSE RESPONSES WITH ENERGY CONSERVATION CONSTRAINTS

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ABSTRACT

This paper considers the problem of constrained tracking the time-varying room impulse response of a source/microphone pair. The constraint which is used to improve the performance stems from the energy conservation that has to hold for real-world impulse responses. We consider three different recursive estimators and compare their performance with the recursive weighted least squares algorithm which does not take the constraint into account. The simulation results show that exploiting this constraint decreases the mean squared error and is thus interesting for applications, especially in the low SNR regime.

Index Terms— Constrained recursive estimation, Room impulse response tracking, Energy conservation

1. INTRODUCTION

In this paper, we study the tracking of a room impulse response (RIR) which is time-varying. In particular, we consider the following setup: Let $\underline{\theta}(n) = [\theta_0(n) \; \cdots \; \theta_{M-1}(n)]^T$ denote the RIR which we want to estimate and let $\underline{s}(n) = [s_0(n) \; \cdots \; s_{S-1}(n)]^T$ be the known signal which is transmitted by the source at time instance n. The corresponding microphone signal $\underline{x}(n) = [x_0(n) \; \cdots \; x_{K-1}(n)]^T$ with length $K = M + S - 1$ is given by

$$\underline{x}(n) = \mathbf{\Theta}(n)\underline{s}(n) + \underline{z}(n) = \mathbf{S}(n)\underline{\theta}(n) + \underline{z}(n) \tag{1}$$

where $\mathbf{\Theta}(n) \in \mathbb{R}^{K\times S}$ and $\mathbf{S}(n) \in \mathbb{R}^{K\times M}$ are Toeplitz matrices that consist of shifted column vectors $\underline{\theta}(n)$ and $\underline{s}(n)$, i.e.

$$\mathbf{\Theta}(n) = \begin{bmatrix} \underline{\theta}(n) & & \mathbf{0} \\ & \underline{\theta}(n) & \\ & & \ddots \\ \mathbf{0} & & \underline{\theta}(n) \end{bmatrix}, \quad \mathbf{S}(n) = \begin{bmatrix} \underline{s}(n) & & \mathbf{0} \\ & \underline{s}(n) & \\ & & \ddots \\ \mathbf{0} & & \underline{s}(n) \end{bmatrix}.$$

Eq. (1) describes a "burst model" where we assume that the RIR is stationary for one input burst $\underline{s}(n)$. The noise $\underline{z}(n)$ in (1) is Gaussian with $\underline{z}(n) \sim \mathcal{N}(\underline{0}, \mathbf{C}(n))$ and temporally uncorrelated, i.e. $\mathrm{E}[\underline{z}(n_1)\underline{z}(n_2)^T] = \mathbf{0}$ for all $n_1 \neq n_2$.

A popular approach in adaptive filtering is the weighted least squares (WLS) estimator which can also be applied to the signal model in (1). The WLS estimator is given by

$$\hat{\underline{\theta}}_{\mathrm{WLS}}(n) = \arg\min_{\underline{\theta}} \sum_{i=0}^{n} \beta^{n-i} \times \left(\underline{x}(i) - \mathbf{S}(i)\underline{\theta}\right)^T \mathbf{C}(i)^{-1} \left(\underline{x}(i) - \mathbf{S}(i)\underline{\theta}\right) \tag{2}$$

where $0 \leq \beta \leq 1$ is the exponential forgetting factor which marks past measurements less valuable than recent ones. This estimator can be efficiently calculated by the recursive weighted least squares (RWLS) algorithm [1] which we will briefly review in Sec. 2.1.

In this paper, we study problem (1) with the additional energy conservation constraint

$$\|\mathbf{\Theta}(n)\underline{s}(n)\|^2 \leq \|\underline{s}(n)\|^2, \tag{3}$$

which states that the energy of the received signal does not exceed the energy of the transmitted signal. Using this additional constraint will allow the design of estimators which have a smaller mean squared error (MSE) than the RWLS algorithm. The recursive estimators that will be derived are the three estimators which we considered in [2]: the recursive constrained maximum likelihood (RCML), the recursive affine minimax (RAMX) and the recursive minimum mean squared error (RMMSE) estimator. They allow the incorporation of general constraints like the energy conservation (3). Thus, the paper gives a particular example for these three estimators.

Beside the many techniques to estimate RIR, which differ in the different choices of excitation signals [3], Lin proposed in [4] two constraints to improve the RIR estimation for the stationary case. He introduced the nonnegativity constraints $\theta_m \geq 0, m = 0, \ldots, M-1$ and added an l_1-norm penalty which controls the sparsity of the solution. It is shown that adding such constraints to the RIR estimation improves the robustness to different noise distributions and decreases the mean squared error. Adding such additional constraints to the RCML, RAMX and RMMSE is also possible and can be done in a straightforward way. Therefore, we will focus in this paper on the energy conservation constraint (3) to improve the tracking performance.

The paper is organized as follows: Sec. 2 briefly reviews the general problem of recursive estimation with constraints and introduces the RCML, RAMX and RMMSE estimators. In Sec. 3, two suitable representations and an approximation of the energy conservation constraint (3) are derived. Sec. 4 then summarizes the RCML, RAMX and RMMSE for our RIR estimation problem and Sec. 5 finally gives some simulation results.

Following notations are used: $\underline{x}$ denotes a column vector, $\mathbf{X}$ a matrix and in particular $\mathbf{I}$ the identity matrix. The trace, matrix transpose and euclidean norm are denoted by $\mathrm{tr}\{.\}$, $(.)^T$ and $\|.\|$, respectively.

2. EFFICIENT RECURSIVE ESTIMATION IN A LINEAR, TIME-VARIANT GAUSSIAN MODEL

In this section, we will briefly summarize three recursive estimators from [2] for a time-variant estimation problem with general constraints. The interested reader is referred to [2] for a more detailed discussion and comparison of the three approaches.

2.1. Signal Model and Sufficient Statistics

Consider the estimation of an unknown, time-variant parameter vector $\underline{\theta}_0(n)$ from observations $\underline{x}(n) \in \mathbb{R}^K$ of a linear, time-varying Gaussian model

$$\underline{x}(n) = \mathbf{S}(n)\underline{\theta}_0(n) + \underline{z}(n), \quad n \geq 0. \tag{4}$$

$\mathbf{S}(n) \in \mathbb{R}^{K\times M}$ is a known model matrix, $\underline{z}(n)$ is Gaussian noise with $\underline{z}(n) \sim \mathcal{N}(\underline{0}, \mathbf{C}(n))$ which is temporally uncorrelated, i.e. $\mathrm{E}[\underline{z}(n_1)\underline{z}(n_2)^T] = \mathbf{0}$ for all $n_1 \neq n_2$, and n denotes the discrete time. In addition, we assume to know a priori $\underline{\theta}_0(n) \in \Theta$, where Θ is an arbitrary subset of $\mathbb{R}^M$. Thus, the task is to estimate $\underline{\theta}_0(n)$ for all $n \geq 0$ subject to $\underline{\theta}_0(n) \in \Theta$ given all observations $\underline{x}(0), \dots, \underline{x}(n)$.

Motivated by the WLS estimator (2), the estimation problem (4) was reformulated in [2] into

$$\underline{x}_n = \mathbf{S}_n\underline{\theta}(n) + \underline{z}_n, \quad \underline{\theta}(n) \in \Theta \tag{5}$$

where $\underline{x}_n$, $\mathbf{S}_n$ and $\underline{z}_n$ are stacked versions of $\underline{x}(i)$, $\mathbf{S}(i)$ and $\underline{z}(i)$ for $i = 0, \dots, n$, respectively:

$$\underline{x}_n = [\underline{x}(n)^T \quad \cdots \quad \underline{x}(0)^T]^T, \; \mathbf{S}_n = [\mathbf{S}(n)^T \quad \cdots \quad \mathbf{S}(0)^T]^T,$$
$$\underline{z}_n = [\underline{z}(n)^T \quad \beta^{-1/2}\underline{z}(n-1)^T \quad \cdots \quad \beta^{-n/2}\underline{z}(0)^T]^T. \tag{6}$$

As the noise $\underline{z}(n)$ is temporally uncorrelated, we have $\underline{z}_n \sim \mathcal{N}(\underline{0}, \mathbf{C}_n)$ where the covariance matrix $\mathbf{C}_n \in \mathbb{R}^{(n+1)K\times(n+1)K}$ is the block diagonal matrix

$$\mathbf{C}_n = \mathrm{diag}\left(\mathbf{C}(n), \beta^{-1}\mathbf{C}(n-1), \cdots, \beta^{-n}\mathbf{C}(0)\right). \tag{7}$$

The unconstrained ML estimator to (5) is identical to the WLS estimator (2) with forgetting factor β. Clearly, $\underline{\theta}(n)$ in (5) is different from $\underline{\theta}_0(n)$ in (4) as (5) is an approximation of the original signal model (4) for a slowly varying parameter $\underline{\theta}_0(n)$. This approximation, however, has proved to be useful in many applications. If $\underline{\theta}_0(n) = \underline{\theta}_0$ is constant, $\hat{\underline{\theta}}_{\mathrm{WLS}}(n)$ will approach $\underline{\theta}_0$ with increasing n. For the case that $\underline{\theta}_0(n)$ changes slowly, $\hat{\underline{\theta}}_{\mathrm{WLS}}(n)$ will follow $\underline{\theta}_0(n)$. The idea is now to use the signal model (5) and derive three estimators which incorporate the constraint $\underline{\theta}(n) \in \Theta$ in different ways.

In order to avoid estimators with a growing computational complexity, the concept of sufficient statistic can be used [5]. A sufficient statistic for the frequentist problem (5) is given by $\underline{t}(\underline{x}_n) = (\mathbf{S}_n^T\mathbf{C}_n^{-1}\mathbf{S}_n)^{-1}\mathbf{S}_n^T\mathbf{C}_n^{-1}\underline{x}_n = \hat{\underline{\theta}}_{\mathrm{WLS}}(n)$. An efficient way to compute it is to use the RWLS algorithm [1]. According to the definition of $\mathbf{S}_n$ in (6) and $\mathbf{C}_n$ in (7), both matrices can be written recursively

$$\mathbf{S}_{n+1} = \begin{bmatrix}\mathbf{S}(n+1)\\ \mathbf{S}_n\end{bmatrix}, \quad \mathbf{C}_{n+1} = \begin{bmatrix}\mathbf{C}(n+1) & \mathbf{0}\\ \mathbf{0} & \beta^{-1}\mathbf{C}_n\end{bmatrix}.$$

This also implies

$$\begin{aligned}\mathbf{R}_{n+1} &= \mathbf{S}_{n+1}^T\mathbf{C}_{n+1}^{-1}\mathbf{S}_{n+1}\\ &= \beta\mathbf{R}_n + \mathbf{S}(n+1)^T\mathbf{C}(n+1)^{-1}\mathbf{S}(n+1).\end{aligned} \tag{8}$$

By applying the matrix inversion lemma $(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{C}^{-1}+\mathbf{D}\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\mathbf{D}\mathbf{A}^{-1}$, we can update $\mathbf{R}_n^{-1}$ and the parameter estimate $\hat{\underline{\theta}}_{\mathrm{WLS}}(n)$ in a time recursive way. Introducing the gain matrix $\mathbf{G}_{n+1} = \mathbf{R}_{n+1}^{-1}\mathbf{S}(n+1)^T\mathbf{C}(n+1)^{-1}$, we finally obtain the RWLS algorithm

$$\begin{aligned}\hat{\underline{\theta}}_{\mathrm{WLS}}(n+1) &= \hat{\underline{\theta}}_{\mathrm{WLS}}(n) +\\ &\quad \mathbf{G}_{n+1}\left(\underline{x}(n+1) - \mathbf{S}(n+1)\hat{\underline{\theta}}_{\mathrm{WLS}}(n)\right),\end{aligned} \tag{9a}$$

$$\mathbf{R}_{n+1}^{-1} = \frac{1}{\beta}\left(\mathbf{R}_n^{-1} - \mathbf{G}_{n+1}\mathbf{S}(n+1)\mathbf{R}_n^{-1}\right). \tag{9b}$$

At each time step, RWLS updates the sufficient statistic $\hat{\underline{\theta}}_{\mathrm{WLS}}(n)$ and the inverse correlation matrix $\mathbf{R}_n^{-1}$. Both values are needed by the recursive estimators below subject to the constraint $\underline{\theta}(n) \in \Theta$.

2.2. Recursive Constrained Maximum Likelihood

The recursive constrained ML (RCML) estimator for (5) is given by

$$\hat{\underline{\theta}}_{\mathrm{RCML}}(n) = \arg\min_{\underline{\theta}\in\Theta}(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))^T\mathbf{R}_n(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n)). \tag{10}$$

At each time step, we have to check $\hat{\underline{\theta}}_{\mathrm{WLS}}(n) \in \Theta$. If this is satisfied, then $\hat{\underline{\theta}}_{\mathrm{RCML}}(n) = \hat{\underline{\theta}}_{\mathrm{WLS}}(n)$. Otherwise, we have to find the minimum of $(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))^T\mathbf{R}_n(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))$ on the boundary of Θ.

2.3. Recursive Affine Minimax Estimation

The recursive affine minimax estimator (RAMX) has the form [2, 6]

$$\hat{\underline{\theta}}_{\mathrm{RAMX}}(n) = (\mathbf{I} + \mathbf{M}(n))\,\hat{\underline{\theta}}_{\mathrm{WLS}}(n) + \underline{u}(n) \tag{11}$$

where $\mathbf{M}(n)$ and $\underline{u}(n)$ are the solution to the minimax problem

$$\min_{\mathbf{M},\underline{u}} \max_{\underline{\theta}\in\Theta} \quad \|\mathbf{M}\underline{\theta} + \underline{u}\|^2 + \mathrm{tr}\left\{(\mathbf{I}+\mathbf{M})\mathbf{R}_n^{-1}(\mathbf{I}+\mathbf{M})^T\right\} \tag{12}$$

and the inverse correlation matrix $\mathbf{R}_n^{-1} = (\mathbf{S}_n^T\mathbf{C}_n^{-1}\mathbf{S}_n)^{-1}$ is updated by the RWLS algorithm. The minimax problem (12) can be rewritten in epigraphic form as shown in (13) at the top of the next page. Since $\mathbf{R}_n^{-1}$ is time-varying, (13) has to be solved repeatedly at each time step.

2.4. Recursive MMSE Estimation

The idea of the recursive MMSE (RMMSE) estimator is to reformulate the frequentist problem (5) to a Bayesian problem where the constraint $\underline{\theta}(n) \in \Theta$ is recast as a priori PDF which is uniform on Θ, i.e.

$$p(\underline{\theta}(n)) = \begin{cases}\text{const} & \underline{\theta}(n) \in \Theta\\ 0 & \text{otherwise}\end{cases}. \tag{14}$$

Such a choice of prior can be motivated by the maximum entropy principle [7]. It states that we should choose that prior which has the most entropy among all distributions fulfilling the constraint $\underline{\theta} \in \Theta$. This is the density given in (14). If, for example, θ is known to lie in the interval $[a, b]$, then the distribution with the maximum entropy is the uniform distribution on $[a, b]$ (see e.g. [8]).

In [2], it is shown that, by using the concept of Bayesian sufficient statistic, the MMSE estimator with the prior (14) is given by

$$\hat{\underline{\theta}}_{\mathrm{RMMSE}}(n) = \frac{\int_\Theta \underline{\theta}\exp\{-\frac{1}{2}(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))^T\mathbf{R}_n^{-1}(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))\}d\underline{\theta}}{\int_\Theta \exp\{-\frac{1}{2}(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))^T\mathbf{R}_n^{-1}(\underline{\theta} - \hat{\underline{\theta}}_{\mathrm{WLS}}(n))\}d\underline{\theta}}.$$

The estimate $\hat{\underline{\theta}}_{\mathrm{RMMSE}}(n)$ is computed using Monte Carlo integration where the samples are generated using rejection sampling [9].

After we have introduced the three recursive estimators that we want to study in this paper, we now turn back to our room impulse response estimation problem which we introduced in Sec. 1.

3. ENERGY CONSERVATION

To improve the estimation performance, we make use of the energy conservation (3) which must hold for real-world impulse responses. We first give two equivalent mathematical representations of (3) and then show how to conveniently approximate them using the discrete Fourier transform (DFT).

3.1. Mathematical Formulation

Expressing the energy conservation mathematically, we need to ensure that[1]

$$\|\mathbf{\Theta}(n)\underline{s}(n)\|^2 = \underline{s}(n)^T\mathbf{\Theta}(n)^T\mathbf{\Theta}(n)\underline{s}(n) \leq \underline{s}(n)^T\underline{s}(n) = \|\underline{s}(n)\|^2$$

[1] We assume in this paper that the source and sensor gains are known, i.e. we have no ambiguity in terms of a scaling of $\mathbf{\Theta}(n)\underline{s}(n)$. This can e.g. be achieved by using a calibration step before estimating the RIR.

$$\min_{\mathbf{M},\underline{u},\tau} \ \tau \quad \text{s.t.} \ \begin{bmatrix}\underline{\theta}\\1\end{bmatrix}^T \begin{bmatrix}-\mathbf{M}^T\mathbf{M} & -\mathbf{M}^T\underline{u}\\ -\underline{u}^T\mathbf{M} & \tau - \operatorname{tr}\left\{(\mathbf{I}+\mathbf{M})\mathbf{R}_n^{-1}(\mathbf{I}+\mathbf{M})^T\right\} - \underline{u}^T\underline{u}\end{bmatrix} \begin{bmatrix}\underline{\theta}\\1\end{bmatrix} \geq 0 \ \forall\, \underline{\theta} \in \Theta \tag{13}$$

which implies

$$\mathbf{\Theta}(n)^T\mathbf{\Theta}(n) - \mathbf{I} \preceq \mathbf{0}. \tag{15}$$

Note that (15) has to hold for all signal lengths S. If the condition is fulfilled for a particular signal length S, it is automatically fulfilled for all smaller signal lengths since an upper-left submatrix of a negative semidefinite matrix is again negative semidefinite. In the following, we will now give two equivalent representations of the set that is described by (15).

(a) LMI representation: Using Schur's lemma [10], we can rewrite (15) into

$$\begin{bmatrix}\mathbf{I} & \mathbf{\Theta}(n)^T\\ \mathbf{\Theta}(n) & \mathbf{I}\end{bmatrix} \succeq \mathbf{0} \tag{16}$$

which is a linear matrix inequality (LMI). Note that (16) immediately implies that (15) describes a convex set.

(b) Frequency domain representation: From above, we know that it is sufficient to consider the case $S \to \infty$. This case can be efficiently computed using the theory of bandlimited Toeplitz matrices [11, 12] as we will now show.

Eq. (15) is equivalent to requiring that all eigenvalues of $\mathbf{\Theta}(n)^T\mathbf{\Theta}(n)$ are smaller or equal to 1. We will therefore now show how the eigenvalues of $\mathbf{\Theta}(n)^T\mathbf{\Theta}(n)$ can be computed for $S \to \infty$. Let

$$r(n) = \begin{cases} \sum_{m=0}^{M-1-|n|} \theta_m(n)\theta_{|n|+m}(n) & |n| \leq M-1\\ 0 & \text{otherwise}\end{cases} \tag{17}$$

be the (unnormalized) auto-correlation function of the RIR $\underline{\theta}(n)$. Then, the matrix $\mathbf{\Theta}(n)^T\mathbf{\Theta}(n) \in \mathbb{R}^{S\times S}$ can be written as a symmetric Toeplitz matrix where the first row is given by $r(n)$ for $n = 0, \ldots, S-1$. Note that $\mathbf{\Theta}(n)^T\mathbf{\Theta}(n)$ is bandlimited as only the first $M-1$ off-diagonals are unequal to zero. Using the asymptotic equivalence of the eigenvalues of a bandlimited Toeplitz matrix and the corresponding circulant matrix, it immediately follows that the condition (15) can be transformed into

$$|\Theta(\omega, n)|^2 = \left|\sum_{m=0}^{M-1} \theta_m(n)e^{-j\omega m}\right|^2 \leq 1 \ \forall\, \omega \in [0, 2\pi) \tag{18}$$

where $\Theta(\omega, n)$ denotes the room frequency response at time instance n. As we consider only real-valued impulse responses $\underline{\theta}(n)$, it is sufficient to restrict ω to $[0, \pi]$ in (18).

3.2. Approximation using DFT

All three estimators will be based on (18) where we evaluate $\Theta(\omega, n)$ at discrete frequencies ω_l using the DFT. Let $\omega_l = \frac{2\pi l}{L}$ with $l = 0, \ldots, L-1$ be the equidistant frequency bins which we consider. $L \geq M$ denotes the DFT length where $L > M$ corresponds to the case of zero-padding. As we only need to evaluate $\Theta(\omega, n)$ in $[0, \pi]$, we have $l = 0, \ldots, \tilde{L}$ with $\tilde{L} = \lfloor L/2 \rfloor$ where $\lfloor x \rfloor$ is the largest integer not greater than x. Using vector notation, we can therefore approximate (18) by

$$\underline{\theta}^T \underline{f}_l \underline{f}_l^H \underline{\theta} \leq 1 \quad \forall\, l = 0, \ldots, \tilde{L} \tag{19}$$

where $\underline{f}_l \in \mathbb{C}^M$ is composed of the first M elements of the lth column of the DFT matrix $\mathbf{U} \in \mathbb{C}^{L\times L}$, i.e. $\underline{f}_l^H = [\ 1\ e^{-j\omega_l} \cdots e^{-j\omega_l(M-1)}\ ]$. Using the DFT, we can therefore approximate $\Theta = \{\underline{\theta}(n) : \underline{\theta}(n) \text{ fulfills (18)}\}$ by the new constraint $\tilde{\Theta} = \{\underline{\theta}(n) : \underline{\theta}(n) \text{ fulfills (19)}\}$ with arbitrary precision if L is chosen large enough.

In the following, we will see that it is convenient to consider the zero-padded vector $\underline{\tilde{\theta}} \in \mathbb{R}^L$ instead of $\underline{\theta} \in \mathbb{R}^M$. Let $\mathbf{P} \in \mathbb{R}^{M\times L}$ be the matrix that consists of the first M rows of the identity matrix. Then, introducing the zero-padded vector $\underline{\tilde{\theta}} = \mathbf{P}^T\underline{\theta}$, condition (19) transforms into

$$\underline{\theta}^T\mathbf{P}\underline{\tilde{f}}_l\underline{\tilde{f}}_l^H\mathbf{P}^T\underline{\theta} = \underline{\tilde{\theta}}^T\underline{\tilde{f}}_l\underline{\tilde{f}}_l^H\underline{\tilde{\theta}} \leq 1 \quad \forall\, l = 0, \ldots, \tilde{L} \tag{20}$$

where $\underline{\tilde{f}}_l \in \mathbb{C}^L$ is the lth column of the DFT matrix $\mathbf{U} \in \mathbb{C}^{L\times L}$, i.e. $\underline{\tilde{f}}_l^H = [\ 1\ e^{-j\omega_l} \cdots e^{-j\omega_l(L-1)}\ ]$ and $\underline{f}_l = \mathbf{P}\underline{\tilde{f}}_l$ holds.

4. ROOM IMPULSE RESPONSE ESTIMATORS

In the following, we will describe in more detail the three recursive tracking algorithms for the estimation of a time-varying RIR.

(a) Recursive CML: The RCML estimator $\hat{\underline{\theta}}_{\text{RCML}}(n)$ for our problem is given by (10) with the constraint (19). This is a quadratically constrained quadratic program which can be solved by standard convex solvers [10].

For completeness, we would like to mention the transform $\underline{\bar{\theta}} = \mathbf{V}^T\underline{\tilde{\theta}}$ where $\mathbf{V} \in \mathbb{R}^{L\times L}$ is an orthogonal matrix. Eq. (21) shows $\mathbf{V}$ for the case that L is even. This transform can be used to simplify the constraints to have the form $\bar{\theta}_0^2 \leq \frac{1}{L}$, $\bar{\theta}_l^2 + \bar{\theta}_{\tilde{L}+l}^2 \leq \frac{2}{L}$ for all $1 \leq l < \tilde{L}$ and $\bar{\theta}_{\tilde{L}}^2 \leq \frac{1}{L}$. Note that the transformation with $\mathbf{V}$ can be efficiently calculated using the fast Fourier transform (FFT).

(b) Recursive AMX: To reduce the computational complexity of RAMX, we consider the special case of $\underline{u} = \underline{0}$ and $\mathbf{M} = \alpha\mathbf{I}$, i.e. we only allow a shrinkage by the factor $1 + \alpha$. Condition (20) is equivalent to

$$\begin{bmatrix}\underline{\tilde{\theta}}(n)\\1\end{bmatrix}^T \begin{bmatrix}-\underline{\tilde{f}}_l\underline{\tilde{f}}_l^H & \underline{0}\\ \underline{0}^T & 1\end{bmatrix} \begin{bmatrix}\underline{\tilde{\theta}}(n)\\1\end{bmatrix} \geq 0 \quad \forall\, l = 0, \ldots, \tilde{L} \tag{22}$$

which has to be fulfilled for all $l = 0, \ldots, \tilde{L}$. Now, the S-procedure can be used to reformulate the optimization problem (13) into a semidefinite program (SDP) [10]. The S-procedure shows that a sufficient condition for the statement

$$\text{for all } \underline{z}\text{: } \underline{z}^T\mathbf{F}_0\underline{z} \geq 0, \ldots, \underline{z}^T\mathbf{F}_{\tilde{L}}\underline{z} \geq 0 \Rightarrow \underline{z}^T\mathbf{G}\underline{z} \geq 0$$

to be true is the existence of $\lambda_0, \ldots, \lambda_{\tilde{L}} \geq 0$ such that $\mathbf{G} \succeq \lambda_0\mathbf{F}_0 + \cdots + \lambda_{\tilde{L}}\mathbf{F}_{\tilde{L}}$. The optimization problem can therefore be rewritten as

$$\min_{\substack{\lambda_0\geq 0,\ldots,\lambda_{\tilde{L}}\geq 0\\ \tau,\alpha}} \tau \tag{23a}$$

subject to

$$\lambda_0\underline{\tilde{f}}_0\underline{\tilde{f}}_0^H + \cdots + \lambda_{\tilde{L}}\underline{\tilde{f}}_{\tilde{L}}\underline{\tilde{f}}_{\tilde{L}}^H \succeq \alpha^2\mathbf{I}, \tag{23b}$$

$$\tau - (1+\alpha)^2\operatorname{tr}\{\mathbf{R}_n^{-1}\} \geq \lambda_0 + \cdots + \lambda_{\tilde{L}}. \tag{23c}$$

Note that (23) is still not a SDP as (23b) and (23c) are not linear in α. However, using the same idea as in [13] and introducing a new variable x and the constraint $x \geq \alpha^2$, we finally obtain a SDP which has the same solution as (23). Furthermore, the constraint (23b) can be simplified by exploiting the fact that $\mathbf{U}^H\underline{\tilde{f}}_l\underline{\tilde{f}}_l^H\mathbf{U} = \mathbf{J}^{l,l}$ where

$$\mathbf{V} = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & \sqrt{2} & \cdots & \sqrt{2} & 1 & \sqrt{2} & \cdots & \sqrt{2} \\ 1 & \sqrt{2}\cos(\omega_1) & \cdots & \sqrt{2}\cos(\omega_{\tilde{L}-1}) & \cos(\omega_{\tilde{L}}) & \sqrt{2}\sin(\omega_1) & \cdots & \sqrt{2}\sin(\omega_{\tilde{L}-1}) \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 1 & \sqrt{2}\cos(\omega_1(L-1)) & \cdots & \sqrt{2}\cos(\omega_{\tilde{L}-1}(L-1)) & \cos(\omega_{\tilde{L}}(L-1)) & \sqrt{2}\sin(\omega_1(L-1)) & \cdots & \sqrt{2}\sin(\omega_{\tilde{L}-1}(L-1)) \end{bmatrix} \tag{21}$$

$\mathbf{J}^{i,j}$ is the single-entry matrix which is all zero except a 1 at position i, j. Hence, we can rewrite (23b) as the new set of constraints $\lambda_l \geq \alpha^2/L$ for all $l = 0, \ldots, \tilde{L}$ and the following SDP is equivalent to (23)

$$\min_{\substack{\lambda_0 \geq 0, \ldots, \lambda_{\tilde{L}} \geq 0 \\ \tau, \alpha, x}} \tau \tag{24a}$$

subject to

$$\lambda_l \geq x/L \quad (l = 0, \ldots, \tilde{L}), \tag{24b}$$

$$\begin{bmatrix} x & \alpha \\ \alpha & 1 \end{bmatrix} \succeq \mathbf{0}, \tag{24c}$$

$$\tau - (1 + 2\alpha + x)\,\mathrm{tr}\{\mathbf{R}_n^{-1}\} \geq \lambda_0 + \cdots + \lambda_{\tilde{L}}. \tag{24d}$$

(c) Recursive MMSE: For the RMMSE, we use directly (18) for the rejection sampling. A FFT of length L is used to calculate $\Theta(\omega, n)$ at discrete frequency points ω_l. Note that (18) describes a convex set. Therefore, we know that the estimate $\hat{\underline{\theta}}_{\text{RMMSE}}(n)$ of the RMMSE will lie in Θ, i.e. it fulfills the energy conservation constraint as was pointed out in [2].

5. SIMULATION RESULTS

To compare the estimators, we consider the problem of estimating the time-varying RIR of a moving source to a fixed microphone. The room size is $3 \times 3 \times 2.3$m. The source moves along a straight line with an increment of 10cm between neighbouring positions. There are $N = 11$ positions which implies that the source moves one meter in total. At each discrete source position n, the RIR $\underline{\theta}(n)$ is calculated using an image source model [14] with the help of the Matlab implementation from [15]. The T_{60} time of the room is 120ms and the sampling frequency is $f_s = 12$kHz. The image source model returned RIRs with $M = 1241$ taps. The noise covariance is $\mathbf{C}(n) = \sigma^2\mathbf{I}$ and the corresponding SNR is defined as $\text{SNR} = 10\log_{10}\|\mathbf{\Theta}(n)\underline{s}(n)\|^2/((M+S-1)\sigma^2)$. To compare the estimators, the normalized error measure $E = 1/N\sum_{n=1}^{N}\|\hat{\underline{\theta}}(n) - \underline{\theta}(n)\|^2/\|\underline{\theta}(n)\|^2$ is used. The DFT length is chosen as $L = 16\,384$ and the signal $\underline{s}(n)$ is a white Gaussian noise of length $S = 100$ such that the total number of observations is $K = M + S - 1 = 1\,340$. The RMMSE estimator is calculated from $I = 3\,000$ samples to approximate the MMSE integrals.

Table 1 shows the simulation results for the four instantaneous estimators WLS, CML, AMX and MMSE, which rely only on $\underline{x}(n)$ and do not take past measurements into account (i.e. $\beta = 0$) and the recursive estimators RWLS, RCML, RAMX and RMMSE. The results are averaged over 200 trials. The optimal forgetting factors are $\beta_{\text{opt}} \approx 0.55$ for a SNR of 5 dB and $\beta_{\text{opt}} \approx 0.28$ for a SNR of 10 dB. The results show that the energy conservation constraint improves the estimation performance, especially for a SNR of 5 dB. If the SNR is higher then all estimators are equivalent, e.g. the RCML does not have to solve for the minimum on the boundary of Θ as $\hat{\underline{\theta}}_{\text{RWLS}}(n) \in \Theta$. The MMSE/RMMSE with a uniform prior shows the best results among all estimators.

6. CONCLUSIONS

In this paper, we compared three recursive estimators for the time-varying estimation of a room impulse response. We incorporated the energy conservation which has to hold for real-world impulse responses into the estimation process and the simulation results show that the estimation error can be decreased, especially in the low SNR regime. The recursive minimum mean squared error estimator with a uniform prior showed the best results.

Note that more a priori knowledge about the room impulse response, e.g. the knowledge that the distance between the source and the microphone will never be below a minimum distance, would allow an even larger improvement compared to the ordinary RWLS algorithm.

7. REFERENCES

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SNR	**WLS**	**CML**	**AMX**	**MMSE**
5 dB	1.23×10^{0}	1.20×10^{0}	1.21×10^{0}	1.02×10^{0}
10 dB	3.88×10^{-1}	3.88×10^{-1}	3.87×10^{-1}	3.72×10^{-1}
SNR	**RWLS**	**RCML**	**RAMX**	**RMMSE**
5 dB	7.02×10^{-1}	6.97×10^{-1}	6.99×10^{-1}	6.52×10^{-1}
10 dB	2.98×10^{-1}	2.98×10^{-1}	2.98×10^{-1}	2.94×10^{-1}

Table 1: Normalized error of the estimated RIRs