Contents

1 Introduction: Signals and Systems **5**

1.1 Signals . . . 5

1.2 Classification of signals . . . 5

1.2.1 Real vs. complex valued . . . 5

1.2.2 Number of channels . . . 5

1.2.3 Dimensionality . . . 5

1.2.4 Continuous vs. discrete independent variable . . . 6

1.2.5 Continuous- vs. discrete-valued . . . 6

1.2.6 Deterministic vs. stochastic . . . 6

1.2.7 More classifications . . . 6

1.3 Some standard signals . . . 6

1.3.1 Unit step function . . . 6

1.3.2 Sign (signum) function . . . 7

1.3.3 Unit (delta) impulse and sifting property . . . 7

1.3.4 Complex exponential functions . . . 9

1.4 Systems . . . 10

2 Analysis of LTI systems in the time domain **15**

2.1 Application of the superposition principle . . . 15

2.2 Step response . . . 15

2.3 Impulse response . . . 16

2.4 Relation between impulse response and step response . . . 16

2.5 Convolution sum (discrete-time LTI systems) . . . 17

2.6 Convolution integral (continuous-time LTI systems) . . . 18

2.7 A stability criterion . . . 20

3 Fourier Transform (continuous-time) **21**

3.1 Introduction . . . 21

3.2 Convergence and Dirichlet Conditions . . . 21

3.3 Transformation of Delta Impulses . . . 22

3.4 Sign Function and Unit Step Function . . . 22

3.4.1 Sign Function . . . 22

3.4.2 Unit Step Function . . . 22

3.5 Properties of the Fourier Transform . . . 23

3.5.1 Reflection . . . 23

3.5.2 Conjugation . . . 23

3.5.3 Symmetry, real/imaginary and odd/even . . . 23

3.5.4 Exchange (time/frequency duality) . . . 23

3.5.5 Linearity . . . 23

3.5.6 Shift . . . 23

3.5.7 Time and frequency scaling (stretch) . . . 23
3.5.8 Convolution . . . 23
3.5.9 Derivatives (differentiation) . . . 24
3.5.10 Integration . . . 24
3.5.11 Central ordinate (initial value) . . . 24
3.5.12 Moment . . . 24
3.5.13 Parseval's Relation and Energy Density . . . 25
3.6 Real Causal Function . . . 25
3.7 Time and Bandwidth . . . 26
3.8 Description of Linear Time-Invariant Systems in the Frequency Domain . . . 28
3.8.1 Frequency Response . . . 28
3.8.2 Phase Delay and Group Delay . . . 29
3.9 Sampling . . . 30
3.9.1 Introduction . . . 30
3.9.2 Fourier Transform of Sampled Signals . . . 31
3.9.3 Sampling Theorem for Low-pass Limited Signals . . . 32
3.10 Reference: Fourier Transform . . . 33
3.10.1 Definition . . . 33
3.10.2 Properties . . . 33
3.10.3 Some transformation pairs . . . 34

4 Discrete Time Fourier Transform 35
4.1 Introduction and basic properties . . . 35
4.1.1 Definition . . . 35
4.1.2 Convergence . . . 35
4.1.3 Periodicity . . . 35
4.2 Delta Impulses in the Frequency Domain . . . 35
4.3 Normalized and denormalized time and frequency . . . 35
4.4 Reference: Discrete Time Fourier Transform . . . 36
4.4.1 Definition . . . 36
4.4.2 Properties . . . 36
4.4.3 Some transformation pairs . . . 37

5 Description and analysis of discrete-time LTI systems 38
5.1 Block diagram . . . 38
5.2 Difference equation . . . 38
5.3 First values of the impulse response from difference equation . . . 39
5.4 Frequency response . . . 39
5.5 Types of difference equations and corresponding systems . . . 40
5.5.1 Non-recursive difference equation and FIR system . . . 40
5.5.2 Purely recursive difference equation . . . 40
5.5.3 General recursive difference equation . . . 40

6 The z transform and its applications **41**

6.1 Introduction . . . 41

6.1.1 Motivation . . . 41

6.1.2 Definition . . . 41

6.1.3 Convergence . . . 41

6.2 Properties of the z transform . . . 42

6.2.1 Linearity . . . 42

6.2.2 Reflection (time reversal) . . . 43

6.2.3 Time shift . . . 43

6.2.4 Stretch in z plane . . . 43

6.2.5 Differentiation in z domain . . . 43

6.2.6 Moment . . . 44

6.2.7 Convolution in time domain . . . 44

6.3 Inverse z transform of rational functions by partial fraction expansion . . . 44

6.3.1 General motivation . . . 44

6.3.2 Partial fraction expansion with polynomials in z . . . 45

6.4 The z transfer function . . . 46

6.5 One-sided z transform . . . 47

6.5.1 Definition . . . 47

6.5.2 Time shift . . . 47

6.5.3 Solving difference equations with initial conditions using z transform . . . 47

6.6 Reference: (two-sided) z transform . . . 49

6.6.1 Definition . . . 49

6.6.2 Properties . . . 49

6.6.3 Some transformation pairs . . . 49